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DETERMINATION OF THE OPTIMUM FREQUENCY OF GAS VELOCITY FLUCTUATIONS IN MOTION AND HEAT EXCHANGE OF PARTICLES

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In a low-frequency region an analytical expression has been obtained for the optimal frequency of gas-velocity fluctuations at which a maximum intensity of heat exchange between a gas and particles is attained. A comparison between the values of the frequency of gas velocity fluctuations obtained from the proposed expression with the results of numerical simulation for various diameters of particles is given.

Keywords: *dynamics of particles, heat exchange, oscillations of particles and gas.*

Introduction. At the present time, increase in the cost of energy along with the necessity of decreasing the cost of production and increasing competitiveness make the problem of energy saving pressing, especially in energy-consuming productions including the processes of heat and mass exchange with disperse systems: heat treatment, drying, combustion, dissolution, etc. One of the developing trends in attaining energy-efficient heat and mass transfer is the creation of optimum nonstationary discrete-pulsed modes of energy input into disperse systems, wave and resonance regimes of the carryier-phase flow with finite amplitude of velocity and pressure fluctuations. In the technique the oscillations of the carryier medium are created by various facilities. Among the efficient generators of high-temperature highly pulsating gas flows are intermittent combustion chambers. Such nonstationary flows can be used for realizing energy-efficient technologies of drying and thermal treatment of disperse materials and solutions [1–7].

Statement of the Problem. During motion of particles in a pulsed gas flow it is important to know the optical amplitude-frequency characteristics of a gas flow that lead to a maximum intensity of heat- and mass transfer. We will consider a one-dimensional motion of a solid (liquid) particle in a pulsed gas flow in the direction opposite to the action of the gravity force, provided the gas velocity changes according to the harmonic dependence

$$v = \overline{v} + v^{\mathrm{a}} \sin (2\pi f t) . \tag{1}$$

Let us consider the case where the hydrodynamic drag force greatly exceeds the Basset force, the force of additional masses, and of the force caused by the pressure gradient. The equation of motion of a particle in an ascending pulsed gas flow in the simplest case of allowing only for the gravity force and hydrodynamic drag has the form

$$\frac{dw}{dt} = -g + \frac{3}{4}\frac{\xi}{d}\frac{\rho_1}{\rho_2}\left| v - w \right|^2 . \tag{2}$$

At $t = 0$ the initial condition is $w = 0$.

It is known that with increase in the relative velocity of the motion of phases the heat- and mass transfer intensity increases. We will determine the condition under which the relative velocity of phases $\left| v - w \right|$ has a maximum.

Analytical and Numerical Solutions. In a pulsed gas flow with a low frequency and finite amplitude of gas velocity fluctuations the gas displacement amplitude l in a wave is much greater than the diameter d of solid particles ($l/d >> 1$), and the process of flow around them can be considered quasi-stationary, i.e., the field of gas velocities at each instant of time obeys the laws governing a stationary flow around bodies. The drag coefficient of a particle can be determined from the equation [8]

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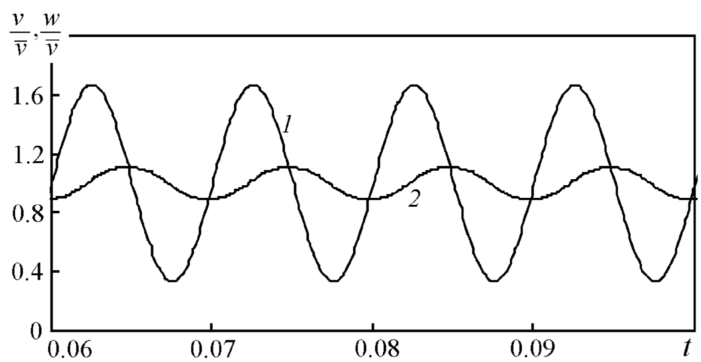

Fig. 1. The velocity of the gas (1) and particles (2) vs. time (v^a = 40 m/sec, $d = 1 \cdot 10^{-4}$ m). t, sec.

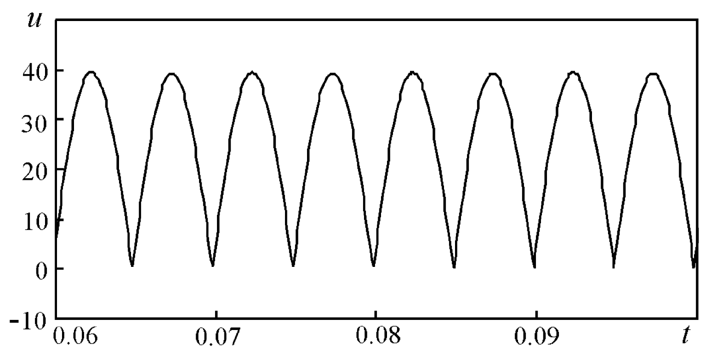

Fig. 2. Relative velocity of phases vs. time. t, sec.

$$\xi = 0.4 + \frac{40}{\mathrm{Re}}, \quad \mathrm{Re} = \frac{d\,|v - w|}{\nu}. \tag{3}$$

Substituting Eqs. (1) and (3) into Eq. (2) and transforming the latter, we obtain

$$\frac{du}{dt} = 2\pi f v^{a} \cos(2\pi f t) + g - \frac{3\rho_1}{10 d \rho_2} u^2 - \frac{30 \nu \rho_1}{d^2 \rho_2} u, \tag{4}$$

where $u = v - w$; $|u| = |v - w|$ is the absolute value of the relative velocity of phases (gas, particle), m/sec.

At the crests of the dependence of the relative velocity of phases on time, the relative velocity of phases acquires a maximum value $|u| = u_{\max}$. This is clearly seen from the dependences of the velocities of the gas and particles on time (Fig. 1) and of the relative velocity of phases on time (Fig. 2) obtained by the method of numerical simulations. Therefore the relative velocity of the motion of phases will be maximal if $du/dt = 0$, then

$$2\pi f v^{a} \cos(2\pi f t) + g - A u^2 - B u = 0, \tag{5}$$

where $A = 3\rho_1/(10 d \rho_2)$ and $B = 30 \nu \rho_1/(d^2 \rho_2)$.

Since $A > 0$ and $B > 0$, the quadratic equation at $u > 0$ has one positive root defined by the expression

$$u = \frac{-B + \sqrt{B^2 + 4A\,(2\pi f v^{a} \cos(2\pi f t) + g)}}{2A}. \tag{6}$$

The maximal relative velocity of the motion of phases is equal to $u_{\max} \approx v^{a}$, when the particles practically fail to follow the gas flow due to their inertia and to the gas velocity fluctuations. With allowance for this condition, from

Eq. (6) we find an approximate computational frequency of gas velocity fluctuations at which the following maximal relative velocity of phases is attained:

$$f_{\mathrm{pr}} = \frac{1}{2\pi}\left(\frac{3\rho_1 v^{\mathrm{a}}}{10d\rho_2} + \frac{30\nu\rho_1}{d^2\rho_2} - \frac{g}{v^{\mathrm{a}}}\right). \tag{7}$$

In Eq. (7) the lower value of the gas velocity amplitude is restricted by the condition

$$v^{\mathrm{a}} > \left(\sqrt{\left(\frac{30\nu\rho_1}{d^2\rho_2}\right)^2 + \frac{6\rho_1 g}{5d\rho_2}} - \frac{30\nu\rho_1}{d^2\rho_2}\right) \Big/ \frac{3\rho_1}{5d\rho_2}\,. \tag{8}$$

This is due to the fact that the aerodynamic drag force of a particle must be greater than the gravity force acting on the particle (the calculated frequency cannot admit negative values). The upper value of v^{a} is limited by the necessity of additional account for the gas compressibility when the velocity of sound is approached.

Analogously an expression for the predicted frequency of gas velocity fluctuations for a descending gas flow was obtained:

$$f_{\mathrm{pr}} = \frac{1}{2\pi}\left(\frac{3\rho_1 v^{\mathrm{a}}}{10d\rho_2} + \frac{30\nu\rho_1}{d^2\rho_2} + \frac{g}{v^{\mathrm{a}}}\right). \tag{9}$$

In [3], an equation of motion of particles in a pulsed gas flow was obtained with allowance for the action of forces attributable to the pressure gradient difference in the velocities and densities of phases, to the gravity force, to hydrodynamic drag, to the force of additional masses because of inertia effects, and to the "hereditary" Basset force appearing due to the nonstationary effects in the carrying phase (the nonstationarity of the boundary layer around particles):

$$\frac{dw}{dt} = -\frac{2(\rho_2 - \rho_1)\varepsilon}{2\rho_2\varepsilon + \rho_1} g + \frac{2\varepsilon\rho_1 + \rho_1}{2\rho_2\varepsilon + \rho_1}\frac{dv}{dt} + \frac{3}{2}\frac{\xi}{d}\frac{\rho_1}{2\rho_2\varepsilon + \rho_1}\,|v - w|\,(v - w)$$

$$+ \frac{18}{\pi d}\frac{\sqrt{2\pi\rho_1\mu f}}{2\rho_2\varepsilon + \rho_1}\,[v - v(0) - w + w(0)]\,. \tag{10}$$

At $\varepsilon = 1$ Eq. (10) describes the motion of a single particle.

The equation of the kinetics of the heating of a particle is

$$\frac{dT_2}{dt} = \frac{6\lambda_1 \mathrm{Nu}}{\rho_2 c_2 d^2}(T_1 - T_2)\,. \tag{11}$$

The Nusselt number was determined from the dependence $\mathrm{Nu} = 2 + 0.55\mathrm{Re}^{0.5}\mathrm{Pr}^{0.33}$. The average Nusselt number is $\overline{\mathrm{Nu}} = \frac{1}{\Delta t}\int_{\Delta t} \mathrm{Nu}\, dt$.

The differential equations were solved by the Runge–Kutta method. The basic initial parameters were: $\overline{v} = 60$ m/sec; $\rho_1 = 0.746$ kg/m^3; $\rho_2 = 1800$ kg/m^3; $T_1 = 473$ K; $T_{2,0} = 293$ K; $\varepsilon_1 = 0.99$, and $c_2 = 1200$ J/(kg·K).

Discussion of Results. The dependences of the average Nusselt number on the frequency of gas velocity fluctuations are given in Fig. 3. It is seen that with increase in the frequency, $\overline{\mathrm{Nu}}$ increases up to a certain limiting value and thereafter practically remains constant. With increase in the diameter of particles the limiting frequency decreases. This is due to the fact that smaller particles "more actively" follow the gas flow, and the relative velocity decreases.

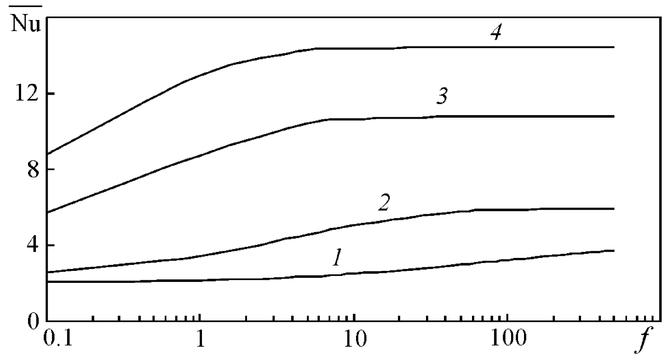

Fig. 3. Nusselt number vs. the frequency of gas velocity fluctuations (v^{a} = 40 m/sec): 1) $d = 2\cdot10^{-5}$ m; 2) $1\cdot10^{-4}$ m; 3) $5\cdot10^{-4}$; 4) $1\cdot10^{-3}$. f, Hz.

For the same parameter of the gas and particles the values of the frequency of the gas velocity fluctuations f_{pr} were determined from Eq. (7). Some of the examples are to be given. The predicted frequency is f_{pr} = 1 Hz for particles of diameter $d = 1\cdot10^{-3}$ m (curve 4 in Fig. 3); f_{pr} = 2 Hz for $d = 5\cdot10^{-4}$ m (curve 3); f_{pr} = 15 Hz for $d = 1\cdot10^{-4}$ m (curve 2). It is seen that the predicted frequencies of gas velocity fluctuations f_{pr} correspond to minimum values obtained by the method of numerical simulation at which the Nusselt number practically ceases to depend on frequency.

Conclusions. The obtained equations (7) and (9) can be used to calculate the optimum frequency of gas velocity fluctuations, with a decrease of which the heat exchange between the gas and particles decreases, whereas with its increase it increases insignificantly.

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NOTATION

c, heat capacity, J/(kg·K); d, diameter of particles, m; f, frequency, Hz; g, free fall acceleration, m/sec^2; l, amplitude of gas phase displacement, m; Nu, Pr, Re, Nusselt, Prandtl, and Reynolds numbers; t, time, sec; T, temperature, K; v, gas velocity, m/sec; w, velocity of particles, m/sec; ε, porosity; λ, thermal conductivity coefficient, W/(m·K); μ, ν, dynamic and kinematic viscosities of gas, Pa·sec and m^2/sec; ξ, coefficient of hydrodynamic drag; ρ, density, kg/m^3. Subscripts and superscripts: a, amplitude; max, maximal; pr, predicted value; 0, initial value; 1, 2, gas and solid particles, respectively; overbar, mean value.

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